

Solutions To Problem Set #4

1. Noether's Theorem says a quantity is conserved if $[H, \mathcal{O}] = 0$ where \mathcal{O} is the operator for the quantity. Here

$$H = p^2/2m + Fz \quad (F = \text{const})$$

$$\therefore [H, p_x] = 0$$

$$[H, p_y] = 0$$

$$\text{but } [H, p_z] = F [p_z, z] = -\hbar F \neq 0$$

$$\text{also, } L_x = \frac{\hbar}{2\pi} (y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y})$$

$$L_y = -\frac{\hbar}{2\pi} (z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z}) \quad L_z = -\frac{\hbar}{2\pi} (x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x})$$

We see that L_x and L_y have $\frac{\partial}{\partial z}$ terms and therefore won't commute with H , but L_z will

So, p_x , p_y and L_z are conserved
 L_x , L_y and p_z are not

\Rightarrow neither L^2 or p^2 are conserved.

2 a) \vec{S} = axial vector (by analogy with \vec{L})

b) \vec{r} is a vector ($\vec{v} \rightarrow -\vec{v}$)

d) $\vec{\Gamma} = \vec{r} \times \vec{p} \rightarrow (-\vec{r}) \times (-\vec{p}) = \vec{r} \times \vec{p}$
 \rightarrow axial vector

e) \vec{p} is a vector since $= m \frac{d\vec{r}}{dt}$

e) \vec{B} can be determined using Biot-Savart:
 $B = \frac{\mu_0}{4\pi} \int \frac{I d\vec{r} \times d\vec{r}'}{r^2}$

\rightarrow axial vector

$$f) \vec{\mu} = \frac{1}{2c} \int \vec{r} \times \vec{j} dV$$

$$\rightarrow \frac{1}{2c} (-1)^2 \int \vec{r} \times \vec{j} dV \quad \text{axial vector}$$

$$g) -\vec{\mu} \cdot \vec{B} \rightarrow -(+\vec{\mu}) \cdot (+\vec{B}) \quad \text{scalar}$$

$$h) (\vec{P}_1 \times \vec{P}_2) \cdot \vec{S} \rightarrow ((-\vec{P}_1) \times (-\vec{P}_2)) \cdot \vec{S}$$

$$i) (\vec{P}_1 \times \vec{P}_2) \times \vec{P}_3 \xrightarrow{\text{scalar}} ((-\vec{P}_1) \times (-\vec{P}_2)) \times (-\vec{P}_3)$$

$$= -(\vec{P}_1 \times \vec{P}_2) \times \vec{P}_3 \quad \text{vector}$$

3. a) If \hat{n} is in the production plane, we can write it as:

$$\hat{n} = c_1 \vec{P}_1 + c_2 \vec{P}_2 \quad c_1, c_2 \text{ real \#}'s$$

$$\text{Then } (\vec{S}_N \cdot \hat{n}) = c_1 (\vec{S}_N \cdot \vec{P}_1) + c_2 (\vec{S}_N \cdot \vec{P}_2)$$

Notice $\vec{S} \cdot \vec{P}$ is a pseudoscalar (see problem 2). Noether's Theorem tells us that if parity is conserved, the expectation value of all pseudoscalars must be 0 since $\langle \vec{S} \cdot \vec{n} \rangle \rightarrow -\langle \vec{S} \cdot \vec{n} \rangle$

$$\text{So } \langle \vec{S}_N \cdot \hat{n} \rangle = 0$$

Not a possible polarization for strong interactions

b) \hat{n} out of production plane means $\hat{n} = c (\vec{P}_1 \times \vec{P}_2)$. This is a scalar

So, it can be non-zero without leading to parity violation

\Rightarrow possible form for polarization

4. Perkins 3.2

$$K^0 \rightarrow 2\pi^0$$

a) π^0 has spin 0 ~~antisymmetric~~
 Bose statistics \Rightarrow symmetric under
 interchange $\Rightarrow (-1)^l = \text{even} \Rightarrow l = \text{even}$

b) no restriction on parity since $K \rightarrow 2\pi^0$
 is a weak decay (changes strangeness)

5. Perkins 3.3

$$\pi^+ \pi^- \text{ with } l=0$$

If we include isospin, $\pi^+ \& \pi^-$ are
 identical particles \Rightarrow Bose statistics tell
 us wave fn is symmetric

$l=0 \Rightarrow$ symmetric under spatial inter-change
 \therefore must be symmetric under isospin

(π has spin = 0)

\therefore under parity $(-1)^2$ for intrinsic parity
 and $(-1)^l$ under parity = +1. This
 is the same as interchanging the $\pi^+ \& \pi^-$
 C then changes them back so

$$CP = +1$$

$\pi^+ \pi^- \pi^0$: $CP (\pi^+ \pi^-) = + (\pi^+ \pi^-)$
 then $CP (\pi^0) = -1$ (due to fact
 that $C(\pi^0) = +1$ $P(\pi^0) = -1$)

$\therefore CP \pi^+ \pi^- \pi^0 = -1$ if $l=0$

6 Perkins 3.5

$$\begin{array}{l} \pi^- p \rightarrow K^0 \quad \Sigma^0 \\ I \quad 1 \quad 1/2 \quad 1/2 \quad 1 \\ I_3 \quad -1 \quad 1/2 \quad -1/2 \quad 0 \end{array}$$

$$|1-1\rangle |1/2 \ 1/2\rangle = \sqrt{1/3} |3/2, -1/2\rangle - \sqrt{2/3} |1/2, -1/2\rangle$$

$$|10\rangle |1/2, -1/2\rangle = \sqrt{2/3} |3/2, -1/2\rangle + \sqrt{1/3} |1/2, -1/2\rangle$$

$$\begin{array}{l} \pi^+ p \rightarrow K^+ \quad \Sigma^+ \\ I \quad 1 \quad 1/2 \quad 1/2 \quad 1 \\ I_3 \quad -1 \quad 1/2 \quad 1/2 \quad -1 \end{array}$$

$$|1-1\rangle |1/2 \ 1/2\rangle = \sqrt{1/3} |3/2, -1/2\rangle - \sqrt{2/3} |1/2, -1/2\rangle$$

$$\begin{array}{l} \pi^+ p \rightarrow K^+ \quad \Sigma^+ \\ I \quad 1 \quad 1/2 \quad 1/2 \quad 1 \\ I_3 \quad 1 \quad 1/2 \quad 1/2 \quad 1 \end{array}$$

$$|11\rangle |1/2 \ 1/2\rangle = |3/2 \ 3/2\rangle$$

If $I = 3/2$ dominates

$$\text{rates go as } \frac{2}{9} : \frac{1}{9} : 1$$

If $I = 1/2$ dominates

$$\text{rates go as } \frac{2}{9} : \frac{4}{9} : 0$$